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# BOOK REVIEWS

**Systems of Conservation Laws 1: Hyperbolicity, Entropies, Shock Waves.** By D. Serre (translated by I. N. Sneddon). Cambridge University Press, 1999. 263 pp. ISBN 0 521 58233 4. £40.

Nonlinear hyperbolic partial differential equations (PDEs) appear in a wide range of models for fluid dynamics. The most well-known areas where such equations arise are compressible fluid flow including gas dynamics and acoustics, and shallow water fluid flow, with the hydraulic jump as the analogue of a shock wave. There are many other ways in which hyperbolic PDEs arise as well: in models for physiological fluid flow (Pedley 1984), in Whitham's (1974) modulation theory for nonlinear water waves, atmospheric flows, magnetohydrodynamics, and in non-Newtonian flows, for example.

The greatest mathematical difficulty with analysing this class of equations is the presence of shocks, which appear as discontinuities, rendering the framework of classical smooth solutions problematic. But shocks are also one of the features of these models of greatest interest in the physical applications! While Riemann's theory and the Rankine–Hugoniot conditions for shocks in one space dimension have been known for well over a century, the range of problems where they were useful was severely limited, and it was not until the middle of this century that progress on such topics as weak solutions, existence of solutions for long times, shock interactions and how to make sense of many discontinuities/shocks (such as occur in combustion problems), and the multi-dimensional case (both the number of state variables and the number of space dimensions) was made.

The book under review is a translation into English by I. N. Sneddon of the book *Systèmes de Lois de Conservation I: Hyperbolicité, Entropies, Ondes de Choc*, originally published in French in 1996. This book is a very readable account of the mathematical tools that have been developed in the past fifty years for this class of PDEs. The author starts by listing many of the models in fluid dynamics, solid mechanics, plasma physics, traffic flow, etc. that can be represented by nonlinear hyperbolic PDEs. He then treats three fundamental topics that are building blocks for the general treatment: (*a*) scalar equations in one space dimension of the form $u_t + f(u)_x = 0$ for $x \in \mathbb{R}$ and $t > 0$ with prescribed initial data, including the Riemann problem for this equation; (*b*) basic properties of linear and quasilinear systems, including local existence properties of the Cauchy problem, weak solutions and a generalization of the Rankine–Hugoniot conditions to systems; and (*c*) the Riemann problem for systems in one space dimension of the form $\boldsymbol{u}_t + \boldsymbol{f}(\boldsymbol{u})_x = 0$ where $\boldsymbol{u} \in \mathbb{R}^n$, with careful attention to the example of gas dynamics. (The Riemann problem consists of solving the Cauchy problem for this system with initial data: $\boldsymbol{u}_0(x) = \boldsymbol{u}_L$ for $x < 0$ and $\boldsymbol{u}_0(x) = \boldsymbol{u}_R$ for $x > 0$ where $\boldsymbol{u}_L \in \mathbb{R}^n$ and $\boldsymbol{u}_R \in \mathbb{R}^n$ are constant vectors.) These three topics, which occupy the first 145 pages and chapters 2–4, would be suitable material for a final-year or masters level introduction to nonlinear hyperbolic PDEs.

The introduction of Glimm's scheme in 1965 was a major turning point in the subject. It lead to a suitable definition of 'solution' that could be shown to exist for all time, and gave a framework for accounting for an arbitrary but finite number of discontinuities as a natural part of the solution. However, the method of proof was not easy and moreover involved a combination of functional analysis and a probabilistic argument! More recent work, though, has shown how to prove Glimm's result from a deterministic viewpoint. Chapter 5 gives a relatively accessible account of the Glimm scheme for systems in one space dimension. This scheme has motivated much work since and some of these results are also considered in Chapter 5. For the applied mathematician, this chapter is pretty hard going. However, the study of this scheme is well worthwhile for applied mathematicians and fluid dynamicists precisely because of the ability to handle many discontinuities and shock–shock interactions easily, for example as arise in combustion problems. In fact, a good starting point to convince the reader of the value of this theory is Chorin's (1976) numerical algorithm based on Glimm's method. Not only is Chorin (1976) a good introduction into the heart of Glimm's method, it shows that the numerical scheme based on it – including the probabilistic argument – can describe a complex system of shock-wave and slip-line interactions without introduction of numerical viscosity and does not require any special attention to discontinuites which freely form and interact. A good example of the effectiveness of this algorithm in fluid mechanics is the numerical calculations of Dillingham (1981), in which the Glimm–Chorin scheme is shown to accurately track multiple travelling hydraulic jumps due to trapped water on the deck of a rolling ship, leading to improved physical understanding.

The final two chapters of the book consider the addition of dissipation terms. In Chapter 6, the case of hyperbolic systems perturbed by second-order diffusion is considered, with the emphasis on existence for the Cauchy problem. In Chapter 7 the subject of 'viscosity solutions' is considered; that is, the relation between solutions of the diffusion perturbed systems such as in Chapter 6 in the limit as the diffusion coefficient goes to zero and the solutions of the unperturbed problem.

For the applied mathematician, the book of Whitham (1974) is still an excellent starting point, because of the physical motivation of his discussion of shocks and shock–shock interactions, the accessible discussion of weak solutions, and the geometrical point of view. In fact, my only quarrel with Serre's book is that the emphasis is distinctly analytical and function-space oriented with little geometry. For example issues such as Hamiltonian structure of conservation laws (cf. Benjamin & Bowman 1987) or geometrical shock dynamics (i.e. §§8.3–8.4 in Whitham's book) are not considered. However, when the basic ideas are mastered, the student and researcher will find this book an excellent and readable account of the developments of the latter half of the 20th century, and an introduction to the current active research in this area.

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## SHORT NOTICES

**Propagation and Reflection of Shock Waves.** By F. V. Shugaev & L. S. Shtemenko. World Scientific, 1997. 244 pp. ISBN 9810230109.

This book contains many interesting photographs of shock waves in gases, presumably taken by the authors or their colleagues (although the attribution is not always clear). Unfortunately the authors have not succeeded in incorporating the photographs into an interesting and intelligible account of the fluid dynamical theory required to explain them. In particular, many pages are taken up with elaborate formulae of doubtful value to most readers, which detract from the presentation of ideas in the book.

**Micropolar Fluids: Theory and Applications.** By G. Lukaszewicz. Birkhauser, 1999. 252 pp. ISBN 3 7643 4008 8. DM 148.

The approach taken in this book is axiomatic: a micropolar fluid is defined to be one capable of transmitting stress torques. A constitutive equation is needed for the non-symmetric part of the stress tensor. The phenomenological choice made here is that due to Eringen. No empirical evidence is offered for this choice; indeed no experiment is discussed throughout the book. A highly mathematical approach is taken with the aim of proving existence and uniqueness theorems for micropolar fluid flows in ‘fairly large function spaces’.

**Flowing Through Time: A History of the Iowa Institute of Hydraulic Research.** By C. F. Mutel. IIHR, 1998. 299 pp. ISBN 0 87414 108 7.

This history of the Iowa Institute of Hydraulic Research is from its beginings in 1920 through to the present day. It includes engaging anecdotes about former directors (Nagler, Hunter Rouse, Kennedy), and an indication of the principal objectives and achievements of the research undertaken by the Institute in each decade. There are numerous photographs.

**Combustion: Physical and Chemical Fundamentals, Modeling and Simulation, Experiments, Pollutant Formation, 2nd edn.** By J. Warnatz, U. Maas & R. W. Dibble. Springer, 1999. 299 pp. ISBN 3540 65228 0. £34.00.

This is the second edition of a book that was first published in 1996. At that time the authors promised that, in view of the fact that their work was principally directed at an audience of new postgraduates, about to embark on research in combustion, they would do their best to keep the material in the book fresh and up-to-date. Evidently the authors meant what they said or, rather, wrote, indeed they repeat it in the new Preface (the original Preface is not reprinted) with the remark that ‘we expect [the book] to be updated in a timely fashion’.

It must be said that differences between the first and second editions are not large, since they are confined to some general ‘polishing’ and correcting of the original text, plus more obvious revisions of material in Chapters 2, 6, 15 and 18. Whether these changes are significant enough to warrant the designation ‘second edition’, in the old-fashioned meaning of that phrase, could be debated. With typesetting done via camera-ready copy directly from the authors it means that authors are able to produce revised text much more quickly now than in the past, pre-word-processor, days ... perhaps we now need to replace edition with another word?

The first edition of the book was reviewed in JFM, vol. 348 (1997), p. 377, to which the interested reader is referred. None of the sentiments or opinions expressed in that review need to be revised in light of the present edition, save perhaps to reinforce the view that, within its clearly stated limitations, the book does an authoritative job for the reader.